Interfacial fluctuations near the critical filling transition

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We advance a method to describe the short-distance fluctuations of an interface spanning a wedge-shaped substrate near the critical filling transition. Two different length scales determined by the average distance of the interface from the substrate at the wedge center can be identified. On one length scale, the one-dimensional approximation of A. O. Parry, C. Rascon, and A. J. Wood [Phys. Rev. Lett. **85**, 345 (2000)], which allows one to determine the interfacial critical exponents, is extracted from the full description. On the other scale, the short-distance fluctuations are analyzed by mean-field theory.

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I. INTRODUCTION

The analysis of uniform physical systems usually involves some reduction in the description of the state of the system. Likewise for inhomogeneous systems consisting of two coexisting bulk phases separated by an interface fluctuating in the presence of a substrate. Certain properties of such systems, e.g., those related to adsorption phenomena, can be described conveniently with the help of a single mesoscopic variable, namely, the distance of the interface from the substrate.

In this paper we consider such a system. The substrate has the form of an infinite wedge extending along the y direction with the opening angle 2φ: see Fig. 1. The quasibulk phase adsorbed on the substrate is denoted β while the phase far above the substrate is the α phase. The surface of the substrate is specified by $z=|x|\cot\varphi$ while $\ell(x,y)$ describes the distance of the α-β interface from the substrate measured parallel to the z axis. It was pointed out recently [1–7] that this system may undergo a critical transition in which the distance of the central part of the interface (above the edge of the wedge) becomes indefinitely large while the asymptotic parts of the interface corresponding to $|x|\to\infty$ remain close to the substrate. This interfacial transition is called the filling transition to distinguish it from the wetting transition that may take place on planar substrates [8,9]. Thermodynamically, the filling transition point is located at bulk α-β coexistence and the filling temperature (which depends on the wedge opening angle φ) is denoted by T_φ with $T_\varphi<T_w$, where T_w is the wetting temperature on the planar substrate.

In their recent paper Parry, Rascon, and Wood [10] used a transfer-matrix method to evaluate—among other features—the values of the critical indices associated with the interfacial behavior near the filling transition. For this purpose a further step in the reduction of the description was made. The two-dimensional interface $\ell(x,y)$ was replaced by the one-dimensional midpoint line $\ell(y)\equiv\ell(0,y)$ (see Fig. 1) for which an appropriate Hamiltonian was proposed.

If, however, one is interested in the full two-dimensional structure of the fluctuating interface near the critical filling transition, i.e., also in the short-distance behavior that is not included in the reduced description then—at least in principle—one should go beyond a mean-field analysis.

In this paper we advance a method to describe the two-dimensional interface close to the filling transition in the system with short-ranged forces [6]. We expect that geometry-dependent effects will be important at short distances. Since the midpoint height does not fluctuate too much on the short length scale, the idea is to fix this height at some arbitrarily chosen point and to assume a mean-field profile of the interface in the vicinity of this chosen point (along the x direction). We use a mean-field approximation to describe the ''relative'' fluctuations around the fixed point. The two-point height distribution function for neighboring points consists of two parts: the one-point distribution corresponding to one of the points (or their average height) and the conditional probability distribution in the form of a Gaussian with a position-dependent dispersion. This dispersion does not diverge at the filling transition and may thus turn out to be useful when geometry-dependent observables are considered. The mean-field description then becomes legitimate because by fixing the position of the interface and looking at the conditional distribution, one forces the local fluctuations to be small and so the system is locally outside the critical region.

II. MEAN-FIELD DESCRIPTION

For the interfacial Hamiltonian in the case of a widely open wedge ($\cot\varphi\ll 1$) we adopt the standard form [6,10]

$$H[\ell]=\int dx\int dy\{\Sigma[\nabla(\ell+\lambda|x|)]^2/2+\omega(\ell)-\omega(\ell_\infty)\}$$

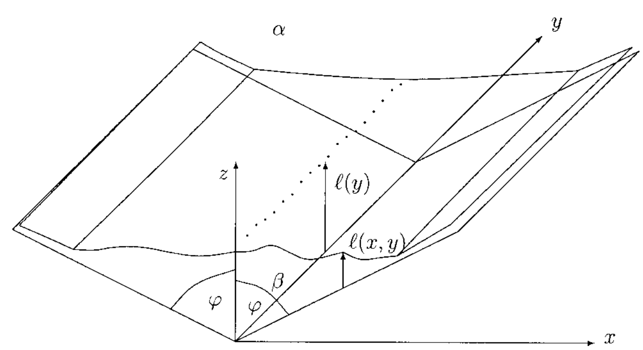

FIG. 1. The wedge geometry and the fluctuating α-β interface.

$$= \int dx \int dy[\Sigma(\nabla\ell)^2/2 + \omega(\ell) - \omega(\ell_\infty)]$$

$$-2\Sigma\lambda \int dy[\ell(0,y) - \ell_\infty], \quad (2.1)$$

where $\ell(x,y)$ (Fig. 1) denotes the thickness of the adsorbed β-like layer measured in the vertical direction while Σ is the α-β interfacial tension. The interfacial pinning potential $\omega(\ell)$ is taken to be that appropriate to critical wetting in the planar case ($\varphi = \pi/2$). For short-range forces, as considered in this paper, it has the form [6–11]

$$\omega(\ell) = -Wt\exp(-\ell/\xi) + U\exp(-2\ell/\xi), \quad (2.2)$$

where ξ is the bulk correlation length (in the β phase) while U and W are positive constants. (We use a convention in which the factor $1/k_BT$ is included in the Hamiltonian.) The parameter t denotes the dimensionless deviation from the wetting temperature for the planar substrate, i.e., $t>0$ for $T<T_w$ and $t=0$ for $T=T_w$. In Eq. (2.1) ℓ_∞ denotes the equilibrium thickness of the adsorbed layer on the planar substrate that minimizes the potential $\omega(\ell)$ so that $\exp(-\ell_\infty/\xi) = (W/2U)t$. Finally, because the wedge is widely open we have put $\sin\varphi = 1$ and written $\cot\varphi = \cos\varphi = \lambda$.

The mean-field profile $\bar{\ell}(x)$ varies only in the x direction. It satisfies the Euler-Lagrange equation [6]

$$\Sigma\bar{\ell}''(x) = \omega'(\bar{\ell}), \quad (2.3)$$

and the boundary conditions $\bar{\ell}(\pm\infty) = \ell_\infty$, $\bar{\ell}'(0_\pm) = \mp\lambda$. The solution of this equation is

$$x(\bar{\ell}) = \pm\int_{\bar{l}}^{\ell_0} \frac{d\ell}{\sqrt{2\{\omega(\ell) - \omega(\ell_\infty)\}/\Sigma}}, \quad (2.4)$$

where the thickness of the mean-field profile at the center of the wedge is denoted by $\ell_0 \equiv \bar{\ell}(0)$. After multiplying both sides of Eq. (2.3) by $\bar{\ell}(x)$ and integrating over x from 0 to ∞ one obtains $\omega(\ell_0) - \omega(\ell_\infty) = \Sigma\lambda^2/2$. With the help of Young's equation one can relate $\omega(\ell_\infty)$ to the contact angle Θ on the planar substrate via

$$-\omega(\ell_\infty) = \Sigma\Theta^2/2. \quad (2.5)$$

From this we see that $\omega(\ell_0) = \Sigma(\lambda^2 - \Theta^2)/2$ while the filling transition ($\ell_0 \to \infty, \ell_\infty \to$ finite) takes place when $\Theta(T = T_\varphi) = \lambda$.

For small deviations, $\delta\ell(x,y) = \ell(x,y) - \bar{\ell}(x)$, from the mean-field profile $\bar{\ell}(x)$, the fluctuation Hamiltonian is

$$H_{fl}[\delta\ell] = H[\bar{\ell} + \delta\ell] - H[\bar{\ell}]$$

$$= \int dx \int d\tfrac{1}{2}[\Sigma(\nabla\delta\ell)^2 + \omega''(\bar{\ell})(\delta\ell)^2], \quad (2.6)$$

i.e., bilinear in $\delta\ell$. An important feature of the critical filling transition is the existence of a *translational mode*, i.e., a fluctuation of the interface that requires very small energy (decreasing to 0 at the transition point). This fluctuation takes the form $\delta\ell(x) = \epsilon|\bar{\ell}'(x)|$ and the corresponding energetic cost is $H_{fl}[\delta\ell] = \epsilon^2\lambda\omega'(\ell_0)$, which decreases to 0 when $\ell_0 \to \infty$.

The corresponding differential equation for the correlation function $G(\mathbf{r},\mathbf{r}') = \langle\delta\ell(\mathbf{r})\delta\ell(\mathbf{r}')\rangle$ has, in mean-field approximation, the following form [12]:

$$[-\Sigma\Delta_{\mathbf{r}} + \omega''(\bar{\ell})]G(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'). \quad (2.7)$$

However, the mean-field description fails in the case of the critical filling transition for short-ranged forces because Eq. (2.7) implies strong anisotropy of fluctuations of the interface: specifically, the fluctuations along the wedge diverge much faster than across the wedge since the latter are bounded by the geometry of the substrate. As shown in [10], the mean-field predictions are valid only for power-law forces of the type $\omega(\ell) \sim \ell^{-p}$ with $p<4$.

III. REDUCTION OF THE ORDER PARAMETER

An *effective* way to analyze the critical fluctuations of $\ell(x,y)$ near the filling transition point is to reduce the interfacial description by looking only at the midpoint height $\ell(y) = \ell(0,y)$ [10]. In order to derive the corresponding Hamiltonian we proceed as follows: first we minimize the Hamiltonian in Eq. (2.1) as in the mean-field method but now subject to the constraint $\ell(0,y) = \ell(y)$ imposed independently at each value of y [13]. From the corresponding Euler-Lagrange equation one obtains

$$x(\ell,y) = \pm\int_{\ell}^{\ell(y)} \frac{d\ell_1}{\sqrt{2\{\omega(\ell_1) - \omega(\ell_\infty)\}\Sigma}}. \quad (3.1)$$

As a result, the one-dimensional Hamiltonian $H_1[\ell(y)] = H[\ell(x,y)]$, valid for configurations given by this expression, takes the form

$$H_1[\ell(y)]$$

$$= \int dy \left\{ \frac{\Sigma[\ell'(y)]^2 \int_{\ell_\infty}^{\ell(y)} d\ell_1 \sqrt{\Sigma(\omega(\ell_1) - \omega(\ell_\infty))/2}}{w(\ell(y)) - \omega(\ell_\infty)} + 2\int_{\ell_\infty}^{\ell(t)} d\ell_1 [\sqrt{2\Sigma\{\omega(\ell_1) - \omega(\ell_\infty)\}} - \lambda\Sigma] \right\}. \quad (3.2)$$

For short-range forces, see Eq. (2.2), this effective Hamiltonian can be evaluated explicitly to yield

$$H_1[\ell]=\int dy\left\{\frac{\Sigma[\overline{\ell}'(y)]^2}{\Theta}\left(\frac{\overline{\ell}-\xi[1-\exp(-\overline{\ell}/\xi)]}{[1-\exp(-\overline{\ell}/\xi)]^2}\right)+2\Sigma(\Theta-\lambda)\overline{\ell}-2\Sigma\Theta\xi[1-\exp(-\overline{\ell}/\xi)]\right\}, \tag{3.3}$$

where $\overline{\ell}(y)=\ell(y)-\ell_\infty$. For temperatures close to the filling transition one has $\ell\gg\xi$ and $\ell\gg\ell_\infty$ and Eq. (3.3) reduces to the one-dimensional Hamiltonian proposed phenomenologically in [10]

$$H_1[\ell(y)]\approx\int dy\left[\frac{\Sigma\ell(y)}{\Theta}[\ell'(y)]^2+2\Sigma(\Theta-\lambda)\ell(y)\right]. \tag{3.4}$$

This Hamiltonian has a relatively simple structure and is easy to renormalize. After introducing the rescaled variables L and Y via

$$\Theta y=(2\Sigma)^{-1/2}[(\Theta/\lambda)-1]^{-3/4}Y,$$
$$\ell=(2\Sigma)^{-1/2}[(\Theta/\lambda)-1]^{-1/4}L, \tag{3.5}$$

H_1 takes the parameter-free form

$$H_1[L(Y)]=\int dY\left[\frac{L(Y)}{2}[L'(Y)]^2+L(Y)\right]. \tag{3.6}$$

Accordingly, the critical behavior of the mean midpoint height $\langle\ell(y)\rangle$ and the correlation length ξ_y follow directly from the rescaling as $\langle\ell(y)\rangle\sim(\Theta-\lambda)^{-1/4}$ and $\xi_y\sim(\Theta-\lambda)^{-3/4}$. These values of the critical indices agree with those obtained in [10].

The one-dimensional system described by the Hamiltonian H_1 in Eq. (3.6) can be solved via a transfer-matrix method [10,15]. However, in this method the presence of the factor $L(Y)$ in front of $[L'(Y)]^2$ is a source of some ambiguity when discretizing the problem and defining the measure that is then used to evaluate the relevant propagator [14]. In order to avoid such problems, it is convenient to introduce the new variable $\eta\equiv 2L^{3/2}/3$ that ''absorbs'' the ''dangerous'' factor $L(Y)$. Then the Hamiltonian takes the form

$$H_1[\eta(Y)]=\int dY\{[\eta'(Y)]^2/2+(3\eta/2)^{2/3}\}. \tag{3.7}$$

The corresponding propagator

$$V(\eta_2,\eta_1,Y)=\int\mathcal{D}\eta\exp(-H_1[\eta])\big|_{\eta(0)=\eta_1}^{\eta(Y)=\eta_2}, \tag{3.8}$$

in which integration over all functions η satisfying the boundary conditions $\eta(0)=\eta_1$ and $\eta(Y)=\eta_2$ is performed, can be evaluated by solving—within the transfer matrix approach [15]—the equation

$$\frac{\partial V}{\partial Y}=\frac{\partial^2 V}{2\partial\eta_2^2}-(3\eta_2/2)^{2/3}V. \tag{3.9}$$

This equation must be supplemented by an appropriate boundary condition when $\eta_2=0$. The general form of such a condition in original ℓ variables, namely,

$$\partial_{\ell_2^{3/2}}V=aV, \tag{3.10}$$

is similar to that found in [15] for two-dimensional (2D) wetting. It follows from Eq. (3.10) that for $a<0$ edge effectively attracts the interface and no filling is observed; the thickness of the β-like layer remains microscopic. Thus we assume $a>0$. Note that $a^{-2/3}$ is the range of the influence of the edge effects. In new variables the parameter η is Θ dependent so one obtains

$$\partial\eta_2\ln V|\eta_2=0=a_\Theta=\tfrac{3}{2}(2\Sigma)^{-3/4}[\Theta/\lambda-1]^{-3/8}a. \tag{3.11}$$

Thus we find $a_\Theta\sim(\Theta/\lambda-1)^{-3/8}$ in the critical region and the appropriate boundary condition is $V(0,\eta_1,Y)=0$.

The propagator $V(\eta_2,\eta_1,Y)$ can be expressed with the aid of the normalized eigenfunctions $\psi_n(\eta)$ and eigenvalues E_n of the equation

$$E_n\psi_n=-\frac{\partial^2\psi_n}{2\partial\eta^2}+(3\eta/2)^{2/3}\psi_n. \tag{3.12}$$

Then one has

$$V(\eta_2,\eta_1,Y)=\sum_n\psi_n(\eta_1)\psi_n(\eta_2)e^{-E_nY}. \tag{3.13}$$

The probability distribution of the midpoint height is given by $\psi_0^2(\eta)$ and other statistical quantities can be expressed by the appropriate combinations of eigenfunctions.

IV. THE SHORT-DISTANCE CORRELATION FUNCTION

Obviously the above one-dimensional approximation cannot describe the full two-dimensional structure of the interface. However, when one considers the correlation function $G(\mathbf{r}',\mathbf{r}'')$, see Eq. (2.7), then for distances $|y''-y'|$ two length scales turn out to be relevant. The one-dimensional character of the filling transition is seen on scales $|y''-y'|\sim\Sigma\ell^3/\lambda$ as shown in the preceding section, see Eq. (3.5), while the two-dimensional structure becomes important when $|y''-y'|\sim\ell/\lambda$ as one expects from mean-field theory. In the *critical region* these two scales are well separated because $\Sigma\ell^2\gg 1$.

Therefore, in order to analyze the short-distance behavior one can introduce *a conditional correlation function*. This is done as follows: We assume that for a certain value of y, say y_0 (for convenience we set $y_0=0$), the interface profile $\ell(x,y)$ is constrained by $\ell(x,y_0=0)=\overline{\ell}(x)$, where $\overline{\ell}(x)$ is described by Eq. (2.4) but with a *given* value of ℓ_0. The full Hamiltonian (2.1) is then expanded in a Taylor series in the variable $\phi(x,y)=\ell(x,y)-\ell(x,0)$ up to terms of order ϕ^2.

In this way one obtains (ignoring a constant term)

$$H[\phi]=-2\int dy\{\lambda\Sigma-\sqrt{2\Sigma[\omega(\ell_0)-\omega(\ell_\infty)]}\}\phi(0,y)+\tfrac{1}{2}\int dx\int dy\{\Sigma(\nabla\phi)^2+\omega''[\overline{\ell}(x)]\phi^2\}. \tag{4.1}$$

The first term on the right-hand side of Eq. (4.1) is small in the critical region (i.e., for $\ell_0\to\infty$ and $\Theta\approx\lambda$). It is of order $2\Sigma(\Theta-\lambda)\phi$, which follows from Eq. (2.5) and the fact that $\omega(\ell_0)\to 0$ for $\ell_0\to\infty$. Thus for short distances (i.e., small compared to $\Sigma\ell^3/\lambda$) from the point at which the constraint is imposed one may keep only the second term in Eq. (4.1). The resulting structure of the Hamiltonian implies that the conditional correlation function

$$G_{\ell_0}(\mathbf{r},\mathbf{r}')=\langle\phi(\mathbf{r})\phi(\mathbf{r}')\rangle|_{\ell(x,0)=\overline{\ell}(x)}, \tag{4.2}$$

obeys the differential equation

$$[-\Sigma\Delta_{\mathbf{r}}+\omega''(\overline{\ell})]G_{\ell_0}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'). \tag{4.3}$$

Similarly, as in Eq. (3.13), the conditional correlation function can be expressed in terms of the normalized eigenfunctions $\psi_{\mathbf{q}}$ and eigenvalues $E_{\mathbf{q}}$ of the operator $[-\Sigma\Delta+\omega''(\overline{\ell})]$ as

$$G_{\ell_0}(\mathbf{r},\mathbf{r}')=\sum_{\mathbf{q}}\frac{\psi_{\mathbf{q}}(\mathbf{r})\psi_q(\mathbf{r}')}{E_{\mathbf{q}}}. \tag{4.4}$$

In this approach one must analyze carefully the contribution from any eigenvalues tending to 0 near the transition. One expects that the eigenfunctions with the lowest eigenvalues will have a structure similar to $\psi_0=|\overline{\ell}'(x)|$, which itself corresponds to the translational mode (although it does not satisfy the appropriate boundary condition in the present case). Thus we introduce the new variables $\psi_{\mathbf{q}}=\varphi_{\mathbf{q}}\psi_0$. The equation for the $\varphi_{\mathbf{q}}$ has the form

$$[E_{\mathbf{q}}+\Sigma\Delta]\varphi_{\mathbf{q}}=-2[\sqrt{2\Sigma\{\omega(\overline{\ell})-\omega(\ell_\infty)\}}]'\partial_s\varphi_{\mathbf{q}}, \tag{4.5}$$

where the prime denotes derivative with respect to $\overline{\ell}$. The expression on the right-hand side of this equation is close to 0 for $\lambda|x|<\ell_0$ while for $\lambda|x|$ approaching ℓ_0 it quickly becomes equal to $-2\sqrt{\Sigma\omega''(\ell_\infty)}\partial_x\varphi_{\mathbf{q}}$, because then $\omega(\overline{\ell})\approx\omega(\ell_\infty)+\omega''(\ell_\infty)(\overline{\ell}-\ell_\infty)^2/2$. We are interested only in long-wave fluctuations such that $E_{\mathbf{q}}\sim\Sigma(\lambda/\ell_0)^2$. If all terms in this equation are to be of the same order of magnitude for $\lambda|x|>\ell_0$, one should have $\partial_x\ln\varphi_{\mathbf{q}}\sim E_{\mathbf{q}}\xi_\pi/\Sigma\sim\xi_\pi\lambda^2/\ell_0^2$, where $\xi_\pi=[\omega''(\ell_\infty)/\Sigma]^{-1/2}$ is the correlation length for the planar case. Note that ξ_π, which diverges at the critical wetting on the planar substrate, remains finite at the critical filling transition.

The above considerations lead to the equations

$$-\Sigma\Delta_{\mathbf{r}}G_{\ell_0}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \tag{4.6}$$

$$\partial_x G_{\ell_0}(\mathbf{r},\mathbf{r}')|_{|x|=\ell_0/\lambda}=0 \tag{4.7}$$

for the conditional correlation function for $\lambda|x|<\ell_0$, i.e., for the central ''free'' part of the interface. One also needs boundary condition when $y'\to\infty$. Since there is long-range order on this scale, i.e., for $\lambda y\sim\ell_0$ one should not expect $G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)\to 0$ when $\mathbf{r}_2\to\infty$. Instead, when $\mathbf{r}_2\to\infty$ we assume $G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)\to f(\mathbf{r}_1)<\infty$, i.e., G remains finite. Using the standard methods of conformal transformation (see the Appendix) one obtains the solution

$$\begin{aligned}G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)=&-\frac{1}{4\Sigma\pi}[\ln\{e^{(Y_1-Y_2)\pi/2}+e^{(Y_2-Y_1)\pi/2}\\&-2\cos(X_1-X_2)\pi/2\}+\ln\{e^{(Y_1-Y_2)\pi/2}\\&+e^{(Y_2-Y_1)\pi/2}+2\cos(X_1+X_2)\pi/2\}\\&-\ln\{e^{(Y_1+Y_2)\pi}+1-2e^{(Y_1+Y_2)\pi/2}\cos(X_1\\&-X_2)\pi/2\}-\ln\{e^{(Y_1+Y_2)\pi}+1+2e^{(Y_1+Y_2)\pi/2}\\&\times\cos(X_1+X_2)\pi/2\}+\pi(Y_1+Y_2)],\end{aligned} \tag{4.8}$$

where $\mathbf{R}_i\equiv(X_i,Y_i)=\lambda\mathbf{r}_i/\ell_0$, $i=1,2$. We note that when $\mathbf{r}_2\to\infty$ one has $G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)\to\lambda y_1/\Sigma\ell_0$.

V. THE SHORT-DISTANCE DISPERSION

For short distances the two-point ℓ distribution may be calculated by constraining one of the points and using the same approximation as in the previous section in which only bilinear terms in the Hamiltonian fluctuations were taken into account. Thus the probability has the form of Gaussian-regularized delta function [16]

$$p(\ell_1,\mathbf{r}_1;\ell_2,\mathbf{r}_2)\approx p(\ell_0)\frac{\exp[-(\ell_2-\ell_1)^2/2\sigma(\mathbf{r}_1,\mathbf{r}_2,\ell_0)]}{[2\pi\sigma(\mathbf{r}_1,\mathbf{r}_2,\ell_0)]^{1/2}}, \tag{5.1}$$

where ℓ_0 is the mean height of the interface above the edge of the wedge defined by

$$\ell_0=[\ell_1+\ell_2+\lambda(|x_1|+|x_2|)]/2\approx\ell_1+\lambda|x_1|\approx\ell_2+\lambda|x_2|. \tag{5.2}$$

We use the conditional correlation function G_{ℓ_0} to obtain an expression for the dispersion σ, namely,

$$\sigma=\langle(\ell_2-\ell_1)^2\rangle=G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_1)-2G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)+G_{\ell_0}(\mathbf{r}_2,\mathbf{r}_2). \tag{5.3}$$

The standard problem that one encounters at this point is that $G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)$ diverges for $\mathbf{r}_2\to\mathbf{r}_1$ [12,17]. This divergence can be removed by regularizing the function $G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)$, e.g., by adding to the Hamiltonian given in Eq. (4.1) a term $\kappa^2(\Delta\phi)^2/2$, where κ is a dimensionless parameter. This procedure yields the equation

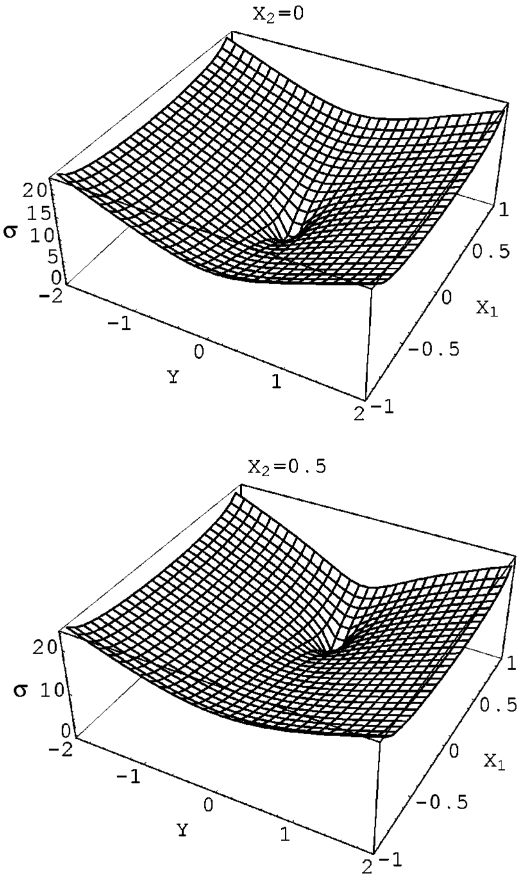

FIG. 2. The dimensionless dispersion σ entering Eq. (5.3) as function of X_1 and Y for $X_2=0$ and 0.5, respectively. Note that $X_i=(\lambda/\ell_0)x_i$, $i=1,2$, and analogous for Y.

$$[\kappa^2\Delta_{r_1}^2-\Sigma\Delta_{r_1}]G_{\ell_0}^{(\kappa)}=\delta(\mathbf{r}_2-\mathbf{r}_1), \tag{5.4}$$

for the regularized function $G_{\ell_0}^{(\kappa)}(\mathbf{r}_1,\mathbf{r}_2)$. For small κ the solution of this equation has the form:

$$G_{\ell_0}^{(\kappa)}(\mathbf{r}_1,\mathbf{r}_2)=G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)-K_0(\Sigma^{1/2}|\mathbf{r}_2-\mathbf{r}_1|/\kappa)/2\pi\Sigma, \tag{5.5}$$

where K_0 is the modified Bessel function. In this way the short-distance divergence is removed and one finds

$$G_{\ell_0}^{(\kappa)}(\mathbf{r}_1,\mathbf{r}_1)=\lim_{\mathbf{r}_2\to\mathbf{r}_1}\left[G_{\ell_0}(\mathbf{r}_1,\mathbf{r}_2)+\frac{\gamma+\ln(\Sigma^{1/2}|\mathbf{r}_2-\mathbf{r}_1|/2\kappa)}{2\pi\Sigma}\right], \tag{5.6}$$

where γ is the Euler constant. Now the expression for the dispersion $\sigma(\mathbf{r}_1,\mathbf{r}_2,\ell_0)$ in Eq. (5.3) can be written down explicitly. We are interested in the situation in which the constraint affects only the mean height of the interface and thus we consider y_1, $y_2 \gg \ell_0/\lambda$ and $|\mathbf{r}_1-\mathbf{r}_2|\gg\kappa$. Then we obtain

$$\begin{aligned}\sigma(\mathbf{r}_1,\mathbf{r}_2,\ell_0)=\frac{1}{2\Sigma\pi}[&2\ln(\ell_0\Sigma^{1/2}/\kappa\pi\lambda)+2\gamma\\&-\ln\{\cos(X_1\pi/2)\}-\ln\{\cos(X_2\pi/2)\}\\&+\ln\cosh\{(Y_1-Y_2)\pi/2\}-\cos\{(X_1\\&-X_2)\pi/2\}+\ln\cosh\{(Y_1-Y_2)\pi/2\}\\&+\cos\{(X_1+X_2)\pi/2\}],\end{aligned} \tag{5.7}$$

where X_i and Y_i are defined below Eq. (4.8). The variation of σ with $\mathbf{r}_1$ at fixed X_2 is shown on Fig. 2. We see that in this limit $\sigma(\mathbf{r}_1,\mathbf{r}_2,\ell_0)$ depends—in addition to X_1 and X_2—only on the distance $Y=Y_2-Y_1$. For fixed values of X_1 and X_2 it is an increasing function of $|Y|$: see Fig. 2. Thus the relative fluctuations of the interface position at points distant along the edge of the wedge become large.

It is interesting to observe that for $|y_1-y_2|\gg\ell_0/\lambda$ one gets

$$\sigma(\mathbf{r}_1,\mathbf{r}_2,\ell_0)\approx|y_1-y_2|\lambda(\Sigma\ell_0)^{-1}/2. \tag{5.8}$$

This result agrees with the prediction of the one-dimensional model that is valid on the scales satisfying $\Sigma\ell_0^3\gg\lambda|y_1-y_2|$; it can be derived with the help of Eq. (3.9). Thus the results obtained via the conditional correlation function in Secs. IV and V are consistent with those stemming from the transfer-matrix analysis of the 1D model in Sec. III.

VI. CONCLUSIONS

A reduced description of a fluctuating interface spanning a wedge-shaped substrate has been derived in an explicit way. This reduced description is based on the one-dimensional Hamiltonian (3.4) [10] and our derivation of this Hamiltonian makes clear the physical assumptions that underlie it. Although the effective one-dimensional Hamiltonian allows one to find the relevant critical exponents, it cannot describe the full two-dimensional structure of the interface. We have proposed a method of supplementing the one-dimensional picture by local, two-dimensional constrained fluctuations that can be analyzed within mean-field theory and described by a conditional correlation function: see Eq. (4.2) These fluctuations are found not to be divergent at the filling transition. The method developed can be used to calculate various other geometry-dependent statistical observables. Explicitly, it predicts the behavior of the dispersion of the conditional correlation function that agrees with the predictions of the one-dimensional model.

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APPENDIX

In this appendix we sketch the successive steps in the method of conformal transformation that lead to the solution of Eqs. (4.6) and (4.7).

After introducing the complex variables $\mathbf{r}_{1,2}=(x_{1,2};y_{1,2})$, $z_{1,2}=x_{1,2}+iy_{1,2}$, Eq. (4.6) can be rewritten as

$$-4\Sigma\,\partial_{z_1}\partial_{\bar{z}_1}G_{\ell_0}(z_1,z_2)=\delta(z_1-z_2), \tag{A1}$$

together with the boundary condition [Eq. (4.7)]

$$i[dz_c\partial_z-d\overline{z_c}\partial_{\bar{z}}]G_{\ell_0}(z,z_2)=0, \tag{A2}$$

where z_c denotes the contour on which the boundary condition is given. These equations are invariant with respect to the conformal transformations. The solution of Eq. (A1) valid for the whole plane has the form

$$G_\infty(z_1,z_2)=-\frac{1}{4\Sigma\pi}[\ln(z_1-z_2)+\ln(\overline{z_1}-\overline{z_2})]. \tag{A3}$$

The solution valid for the semiplane $\Re z>0$ with a Neumann boundary condition on $\Re z=0$ may be found with the help of the method of images and has the form

$$G_{\Re z>0}(z_1,z_2)=G_\infty(z_1,z_2)+G_\infty(z_1,\overline{z_2}). \tag{A4}$$

After introducing the conformal transformation $Z\mapsto e^{-i\pi Z/2}$ for dimensionless variables $Z=z\lambda/\ell$ and $\mathbf{R}=\mathbf{r}\lambda/\ell$, one obtains

$$G_{(\ell)}(Z_1,Z_2)=G_{\Re Z>0}(e^{i\pi Z_1/2},e^{i\pi Z_2/2}), \tag{A5}$$

with the Neumann condition at $X=\pm 1$. Finally, taking into account the constraint imposed on the interface at $y=0$ and using the freedom to add to the right-hand side of Eq. (A4) the solutions of Laplace's equation lead to the solution of Eq. (4.6) given in Eq. (4.8).

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